Riemannian Geometry
as Best Possible Picture of the Material World

by *Ulrich Bruchholz*

Since the mankind exists, this is characterized by the effort to simplifiedly represent the world, and imagine it with simple pictures. As well, they must soon realize that it is *on principle* not possible with the *ideal* world. That means, the question for the „Why" is useless and even harmful. The reason of our existence is not our merit, and can never be clarified. Who claims that for himself, presumes to know more than God. - However, scientists are paid to fathom the world *as is*, and to find well working models for it.

The current imagination consists in it that any „matter" be suspended in the space, and change in the time. This picture is very disturbed by the discovery of the fields. A field stands for a certain property of space and time dependent on the place in space and time. There are fields that propagate in the vacuum ! That are gravitation and electromagnetism. These fields are understandable only as *geometric* properties of space *and* time.

Since we live in space and time, we cannot *directly* see the geometric properties of space and time. But with the three dimensions of the space, we can very well investigate the properties of two-dimensional spaces - that are surfaces. GAUSS has already done that for us. With it, such surface is described at each place of the surface by an only one quantity - the GAUSSian curvature. The GAUSSian curvature is independent on the chosen coördinates and, with it, on the location of the surface in the space.

Bernhard RIEMANN defined a n-dimensional manifold from $n(n-1)/2$ mutually orthogonal surfaces. The properties of these surfaces are known with their GAUSSian curvatures. (These are called RIEMANNian curvatures now.) Example: One can mutually orthogonally put three smooth surfaces at each point in the space. As we know that from EUKLID, these are plane. But what is, if the surfaces bend with this action ? Then the space itself is curved (and conversely).

What is with the time ? Albert EINSTEIN saw that there is no absolute time, but everybody has his own time. With it, the time becomes a *coördinate* and, as such, a geometric category. Hermann MINKOWSKI found that the LORENTZ transformations (which concern the relations of length and time at relative motion) are a simple coördinate transformation, if one introduces the time as fourth coördinate. As well, the time is an imaginary length and conversely. Relation: $1s = j\ 300000$ km (with $j^2 = -1$). That means, our existence is time-like. EINSTEIN told of „four-dimensional threads". - In it, the light is a borderline case, because the light does not become older with its propagation.

After space and time have been unified to a four-dimensional space-time as geometric object, there is the question how the geometric properties of the space-time be to relate to physics. EINSTEIN has found the key also here. - Each point in the space forms a curve in the four-dimensional space-time. This curve is geometrically decided essentially by its *curvature vector*. If we compare NEWTON's force equation with the formula for the curvature vector, results *force = mass times curvature vector* . That means, *both*, accelerated motion *and* gravitation, are parts of the curvature vector.

If the properties of the curves in the space-time are known, one can conclude also the geometry of the space-time itself from them. EINSTEIN found a relation between the RICCI tensor and an energy-momentum tensor. That is a jumble of geometrical and physical quantities. However, one can decide the gravitation from the distributed mass. *And understand as geometric property of the space-time.* - Ok, what is the mass ? In order to find out that, one should turn to the other field in the vacuum, the electromagnetism.

The electromagnetic field is described by the MAXWELL equations. LORENTZ found the energy and momentum components of the electromagnetic field, that one can sum up to an energy tensor. If this energy tensor is inserted to EINSTEIN's gravitation equation, a *purely geometrical* system of equations results ! Under a weighty condition: *Masses and momenta in the Einstein equations as well as charges and currents in the Maxwell equations* ***must*** *vanish !* - Where do these keep then ? The answer to this question is as simple, that it was overlooked up to now. These „material" quantities appear as integration constants of the source-free EINSTEIN-MAXWELL equations. Mass, spin, electric charge, and magnetic momentum are the first integration constants. (There is no answer to the question, why is it so. The conversion results with comparison of the field courses from the phenomenological and from the geometrical equations.) - If certain marginal conditions are present, the integration constants take on discrete values. These involve discrete solutions then.

Though the source-free EINSTEIN- MAXWELL equations can involve discrete solutions, these have, in *seeming* contradiction to it, many degrees of freedom, because there are only 10 independent equations for 14 variables. That means, the world is basically *not* causal. In order to understand this geometrically, we should return to RIEMANN's surfaces.

In general, the space-time should be described by $4{\cdot}3/2 = 6$ mutually orthogonal surfaces. However, a derivation from the source-free EINSTEIN- MAXWELL equations results, that only two dual (these cut in the regarded *point*) surfaces with special curvature relations play a role, and the others (orthogonal to them) do not act. That implies the mentioned degrees of freedom again. The electromagnetic field tensor is expressed from exactly these two surfaces, is so a geometric quantity ! - Newer mathematical works (e.g. from DONALDSON *) prove that just the four-dimensional manifold plays exactly this special role. The space-time is geometrically unique, and no different manifold were able to take on the special geometric properties exactly returning the physical reality.

As for the quantization, it be referred to numerical simulations, in which correlations result that point at significant correspondence of the integration constants for most stable solutions with known particle values. Contexts with the chaos need investigations.

*) according to an information by Werner Mikus, Köln/Germany.

References to this article are to find in http://bruchholz.psf.net/article2.txt .

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