What is Geometric Theory of Fields ?

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31 October 2005

The present physical thinking is dominated by the imagination of "matter" in space and time that determine all events around us. As well, nobody can say what matter be, and physicists make it their business to fathom this.

The effect to a body known for long time is a force at accelerated motion on the one hand, and caused by gravitation on the other hand. With the mass term, Isaac NEWTON has found a measure depending only on the body itself. As well, it occurs as though there be two qualitatively different masses, namely an inert and a heavy mass.

NEWTON could clear up that each mass builds up a gravitation field around itself, that decreases in its effect for the factor $1/r^2$ with the distance. NEWTON has experimentally determined the proportion factor, the gravitation constant, for it too. He took as working hypothesis the assumption that space and time be given forever, and independent on the matter being situated in them. That means a distant action, because the one body works with its mass immediately to the other body with its mass via the gravitation. In the daily life, the dominant body is the earth with its gravitation field. - However, NEWTON himself was never happy with the working hypothesis and the distant action resulting from it. The inertia keeps mysterious as well. NEWTON could only take notice that the second derivative

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of the way, done by the body, to the time is the deciding factor.

The exploration of the electromagnetism involved a great step of discovery. Faraday described the electric and magnetic fields with force lines so called by him. These work only from one point up to the adjacent point, from this up to the next again, and so on as well in the space as in the time too. This action does not jump-like happen, but there are only appropriately small changes from point to point. One can answer the legitimate question for the point distances, that these are chosen *any* small. That is called a continuum, and the action is a *near action* .

An essential consequence from the near action is the prediction of electromagnetic waves by Maxwell, which Heinrich Hertz has experimentally detected first. The light is under them, so that the electromagnetic waves propagate with light speed.

Now, the search for the assumpted medium begun, in which the electromagnetic waves are supposed to propagate. As well, preferably the total independence of the light speed on the observer's relative motion to the light source irritated. But there is no dependence on an absolute motion (e.g. of the earth at the rotation around the sun) too, as the known Michelson experiment demonstrated. With it, the existence of such medium, also known as ether, was questioned at all.

These *seeming* discrepancies made Albert Einstein deal with relative motions. - In mechanics are so called inertial systems moving in straight lines and unacceleratedly. The behaviour of bodies is the same on all inertial systems. That does not work in electrodynamics. Moved charge induces a magnetic field, and a moved magnet an electric field to a resting system. Lorentz designed transformation formulae for the inertial systems, which are supposed to consider the constant light speed. As well, time and length in the moved system are changed for the resting observer. The moved body appears shorter in direction of the motion, and the clocks go slower on the body.

EINSTEIN saw that the LORENTZ transformations return the *real* change of the scales and clocks at relative motion. With it, everybody has his own time. Time and space are nothing absolute but connected with each other via LORENTZ transformations. With this interpretation, the principle of relativity is valid not only in mechanics but also in electrodynamics. The induction laws become a part of transformation relations of the electric and magnetic fields, which are unified this way.

The LORENTZ transformations appear pretty arbitrary with the postulate of the constant light speed alone. We owe Hermann MINKOWSKI the deciding comprehension jump with his geometric interpretation of the time. MINKOWSKI took the time as fourth coördinate (together with the three spatial) by setting $x_4 = \mathsf{j}ct$ with $\mathsf{j}^2 = -1$. Therefore, the time is imaginary length, or a length is imaginary time. Since we live in the time, a line element in the space-time performed this way becomes time-like, i.e. with the real time $\mathsf{d}s^2 = (c\mathsf{d}t)^2 - \mathsf{d}x^2 - \mathsf{d}y^2 - \mathsf{d}z^2 > 0$. However, for the light is $\mathsf{d}s = 0$.
The right of this geometric interpretation of the time is given by the fact that, with the LORENTZ transformations, the time itself becomes a coördinate. The electromagnetic wave equations are a visible certification of it. In the space-time, the LORENTZ transformations themselves become a simple rotation of the time coördinate and the x coördinate (at motion in x direction) for an *imaginary* angle ψ. The relative velocity is the tangent of this angle $v = \mathsf{j}c\tan\psi$ then. Mathematics results from it in the addition theorem of the velocities inclusive the fact that a body can never reach the light speed.

MINKOWSKI has tried to trace these facts on the paper, i.e. the time like a real length. For this, it is referred to the relevant literature under the headword "MINKOWSKI cone". This analogy has its limits, however, the so-called twin paradoxon and similar things can be well understood with it. See also [1].

What are fields ? - EINSTEIN could answer this question for the gravitation via the equivalence principle:

EINSTEIN has identified the *special relativity*, i.e. the relativity of motion of inertial systems (that is valid also for electromagnetism), via the LORENTZ transformations. Consequently, EINSTEIN has been led to the search for a *general* relativity of motion. That means the same as the question for the relativity of accelerated reference systems.

We notice in accelerated reference systems that a force is effective against a massive body. The same force action is in the gravitation field too. If the observer does not *know*, where the force comes from, he cannot notice it. For that reason, the question for the general relativity leads to a new question for the origin of the two force actions. EINSTEIN raised the equivalence of inert mass and of heavy mass to a principle, the *equivalence principle.*

Now, the geometry can help again. -
Each body describes a curve in MINKOWSKI's four-dimensional space-time. For the unmoved body, this curve is identical with the time axis, at unaccelerated motion a straight line inclined to the time axis, as explained in the special relativity. An acceleration leads to a bent curve. The most important parameter of a curve is its curvature vector. (It is the total derivative of the tangent vector.) A comparison of the physical parameters with the curvature vector results in remarkable: NEWTON's force equation $\mathcal{F} = m\,(\frac{\partial^2 \mathcal{X}}{\partial t^2} + \mathcal{G})$ consists of the two equivalent parts. The second derivative of the way to the time is the accelerated motion, during $\mathcal{G}$ summarizingly expresses the field strength of the gravitation. The curvature vector contains the second derivative of the local vector to the distance on the curve, which means the local time of the accelerated body and, with it, is identical with the observer's

time in first approximation. It follows cogently from the physical fact that the curvature vector *must* have a second part. That is the case exactly then, if the space-time itself is curved ! Any curvatures of the space-time go into the parameters of the curve.

The curve in the space-time for *force-free* motion is a geodesic, because the curvature vector vanishes for it. The equation of the geodesic $\mathcal{K} = 0$ is physically an equation of motion with it.

One may understand the curvature of the four-dimensional space-time (not the three-dimensional space alone !) analogously to the curvature of a surface, as known from the daily life. This generalization of the geometry goes back upon Bernhard Riemann.

The line element on a curved surface results with any coördinates on the surface from $\mathsf{d}s^2 = g_{11}(\mathsf{d}x_1)^2 + 2g_{12}\mathsf{d}x_1\mathsf{d}x_2 + g_{22}(\mathsf{d}x_2)^2$. The general relation for many dimensions is then $\mathsf{d}s^2 = \sum_{\mu,\nu} g_{\mu\nu}\mathsf{d}x_\mu\mathsf{d}x_\nu$ with $g_{\nu\mu} = g_{\mu\nu}$.

The coëfficients $g_{\mu\nu}$ perform a symmetric tensor, which completely returns metrics of the manifold. With metrics, each distance in the manifold can be determined. However, metrics essentially depends on the chosen coördinates, and, with it, is no measure for the curvature of the surface respectively the space-time. (However, the curvature goes into metrics !)

Gauss found out that the properties of a surface at each point of the surface are described with a single quantity ! This Gaussian curvature is the product from maximum and minimum *vertical* curvature.

The further above described curvatures are horizontal curvatures, i.e. *in* the surface resp. space-time. A geodesic can be vertically curved. So one can understand, why a sheet of paper can be arbitrarily rolled. The Gaussian curvature of the paper keeps always zero !

Riemann had the brilliant realization that the properties of an n-dimensional manifold can be expressed from the Gaussian curvatures of $n(n-1)/2$ mutually orthogonal surfaces being situated in the manifold. That are 6 surfaces in the space-time, namely the 3 known spatial surfaces, and the 3 surfaces being stretched by the time and one spatial coördinate each.

Above mentioned GAUSSian curvatures are now replaced by parallel shift of vectors, that makes possible the use of the tensor calculus. Tensors are invariant quantities in their entity. The tensor components follow generally valid transformation laws.

This mathematics has been founded by a school of mathematicians under RICCI, LEVI-CIVITÁ, & al. at EINSTEIN's times. EINSTEIN was the first user.

These curvature measures are *not* identical with the gravitation ! However, there is the close context of these curvatures *and, with it,* metrics with the curve parameters.

From the context of gravitation with curvature of the space-time follows that general relativity can be defined only *locally*, i.e. in immediate surroundings of a point in the space-time. We can define EUKLIDian conditions there. But since metrics at gravitation is different from metrics without gravitation, the local scales and clocks behave differently from scales and clocks out of the gravitation field. The clocks tick slower for the outer observer, and the scales become longer. That means for the outer observer a smaller light speed in the gravitation field, what led to EINSTEIN's famous prediction of the bending of rays of light in gravitation fields. However, the *local* light speed is constantly c !

It should be mentioned, that the gravitation field itself contains no energy. However, observers on distant positions can notice different energy states of the same system due to differences in metrics. With it, gravitation does not transfer energy but arranges this.

With the general theory of relativity, we have the paradoxical situation that the gravitation is derived from the geometry of the space-time, during the masses are seen as “generating” gravitation, and the structure of them keeps unsolved. This paradoxon can be only seen in the context with another till then unsolved questions:

- What is electromagnetism ?
- What about the quantization ?

The quantization of physical quantities is a fact of experience, which is manifested also from it, that statistical methods have been successfully used. We have to suppose that these three unsolved questions are to solve only together.

One may hope for reasonable answers to questions to the nature, only if these questions are unbiased. That means, no answers may be expected or even given. In the context with the three unsolved questions, following concrete questions appear useful:

1) What quantities are conserved ?
2) What quantities have discrete values ?

Conserved are the "material" quantities mass, spin, electrical charge, and magnetical momentum. For the equivalence of mass and energy, following from special relativity, the conservation of energy follows from the conservation of mass. Mathematics gives the surprising answer, when these quantities take on discrete values: *as integration constants of source-free partial differential equations !*

That is the case for example with a present boundary, in which one may not expect the classical margin of the potential theory. The concrete circumstances can be found out by means of e.g. numerical simulation.

Mass and spin are the first integration constants of EINSTEIN's gravitation equations, and charge and magnetical momentum are those in MAXWELL's equations. Distributed masses and momenta as well as distributed charges and currents do not exist !

The non-existence of distributed charges and currents becomes clear also from the mesh and knot laws known in electrotechnique, when the meshes and knots become very small. That is no contradiction to the fact, that one can measure a current in a mesh and a voltage between two knots.

On these conditions, EINSTEIN's and MAXWELL's equations can be unified via the energy tensor of the electromagnetic field. The resulting source-free EINSTEIN-MAXWELL equations[1] obtain purely geometric meaning then !

The sources are an equivalent representation of accumulated singularities from the integration constants. However, the energy law is violated with the sources. - Numerical simulations do not lead to singularities, if one accepts the existence of *geometric boundaries*.

The possibility, describing the material quantities as integration constants, is known, however, it is not accepted up to now. Why ? It contradicts the imagination established by Ernst MACH, that "matter" be suspended in the space and (secondarily) generate the fields and, with it, determine the structure of the universe. Actually, this kind of matter is not needed. Matter is manifested in the integration constants. All physical (energy law !) *and* mathematical difficulties are cancelled for the price of the *traditional* matter. The particles are discrete (elementary) solutions of the source-free EINSTEIN-MAXWELL equations then. There are already significant indications from numerical simulations ! [1] -

Within tolerances of $\pm 5\%$, the equivalent integration constants lead to most stable solutions, when the values of them are identical with the measured values of spin, charge, magnetic momentum, and geometric boundaries appear at the presumed particle radius. These four values are mutually conditional !

Though solutions exist only from discrete values of the integration constants, such solution is ambiguous in the near field. The near field has an extension about 10^{-15}m for nuclei, for more complex solutions (atoms, molecules, &c.) essentially more.

With the question, how photons result from the EINSTEIN-MAXWELL

[1] quoted in [1]

equations, it be referred to [1]. There are to find also detailed reports and results from numerical simulations.

We have noticed that in the geometric analogy the gravitation together with the accelerated motion is a curve parameter, namely the curvature vector of the curve in the space-time. The very similar properties of the electromagnetism, mainly the propagation in the vacuum, force to the conclusion that electromagnetism means curve parameters too. That must so be, because each measuring tool describes such curve in the space-time. The entire set of these curves is sufficient to describe the curvature circumstances in the space-time.

During gravitation and accelerated motion perform a vector accompanying the curve, electromagnetism can be described from two accompanying surfaces, which are stretched by two vectors each. - The curvature vector is space-like and directly to feel. The term “below” means nothing else than the *direction* of the curvature vector from the gravitation field of the earth. Unfortunately, the surfaces from electromagnetic fields are to feel not as directly. At this place, it can be said only, that it concerns two dual surfaces with quite special curvature properties (more see [1]). The source-free EINSTEIN-MAXWELL equations involve a special geometry, that is possible exclusively in the four-dimensional space-time. It *is the* geometry of the space-time, and this is unique ! The space-time does not allow another geometry !

The accompanying surfaces manifest the difference between electric and magnetic field, because the one surface is time-like in one dimension. There is no symmetry of electricity and magnetism for it !

The unique geometry of the space-time expresses itself also with the incomplete causality. The source-free EINSTEIN-MAXWELL equations perform only 10 independent equations for 14 variables, what means very many degrees of freedom. In the physical reality, however, the special role of the time becomes essential. Since we live in the time, the world is causal in first approximation. (One can give mathematical reasons for this.) That is not more true in the micro range !

Metrics is not the field itself, however, one can easily understand that metrics is influenced by curvatures of the space-time. Quite practical calculations result in following tendencies:
Gravitation expresses itself *for the distant observer* in it, that the clocks tick slower and the scales become longer. However, electromagnetism goes quadratically into metrics. The clocks tick faster and the scales become shorter, at the electric field in direction of the field strength and at the magnetic field vertically to the field strength.

The shortening of the scales at the electric field has quite practical consequences: With the shorter distance, the field strength becomes greater, with it the distance shorter and shorter and so on. That means a feedback, that leads to a zero-distance between two points with finite distance in the coördinate system. The unbiased reader may wonder, how lightning and tunnel effects with super light speeds come about.

Numerical simulations regularly lead to a geometric boundary, where is no time. Within this boundary is nothing, neither space nor time ! For this reason, it is useless to speculate what may be within a particle.

The author owes interesting suggestions for this work to a discussion circle with the gentlemen Prof. Manfred Geilhaupt, Dr. Gerhard Herres and Werner Mikus . Here are especially to emphasize valuable hints by Werner Mikus, who saw to it that the article is intelligible.

Reference :

[1] Bruchholz, U. : http://bruchholz.psf.net/ .

More references in ./article2.txt
On the special role of the geometry (intelligible) see ./Geometry.pdf
The geometric theory as textbook (only in German) see ./Textbook.pdf
On numerical simulations ./feldber-.htm or ./feldber.zip
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